

Confidence Intervals and Prediction Intervals

Exercise 1. Global Warming and Butterfly Size

Between 1996 and 2013, scientists sampled 32 *Boloria chariclea* butterflies. They recorded the wing length of these butterflies (y_i , in mm) as well as the average temperature of the preceding summer (x_i , in Celsius). They then fit a simple linear regression model (with Normal errors) to the data; the output below shows the first two columns of the corresponding regression summary table:

```
mod_butterfly <- lm(Wing ~ Temp, data = ButterfliesBc)

mod_butterfly |>
  broom::tidy() |>
  mutate(statistic = NA, p.value = NA)
```

```
# A tibble: 2 x 5
  term      estimate std.error statistic p.value
  <chr>      <dbl>     <dbl>   <lg1>    <lg1>
1 (Intercept)  18.9       0.287    NA      NA
2 Temp        -0.235     0.122    NA      NA
```

1. Write down the value of the fitted slope coefficient, $\hat{\beta}_1$. Interpret the slope of this model in context.

2. Calculate the t -statistic for the test of $H_0 : \beta_1 = 0$ vs. $H_A : \beta_1 \neq 0$.

3. Let $\alpha = 0.05$. Use the following information about the left-tail of the t -distribution on 30 degrees of freedom to draw a picture of the null distribution and where the t -statistic falls in it.

```
qt(0.025, df = 30)
```

```
[1] -2.042272
```

4. Is there a statistically significant association between temperature and wing width? Why or why not?

Exercise 2. Types of Intervals

For each of the following scenarios, identify whether the research question would be best addressed by:

- a. a confidence interval for the slope of a regression model
 - b. a confidence interval for a subpopulation mean response value
 - c. a prediction interval for a future, individual response value
1. An up-and-coming musical artist would like to know how much money their new single will make if they release it on Spotify.
 2. A Survivor fan wants to investigate ageism on Survivor, and is particularly interested in whether, and to what extent, older players are eliminated earlier in the game.
 3. A school district in Massachusetts hopes that improving the quality of the school's water will improve their students' test scores; they would like to know the students' average test score if they drink water with 5 parts per billion (ppb) lead. (The Environmental Protection Agency action level for lead in drinking water is 15 ppb.)
 4. A Hollywood executive is interested in the typical earnings of a movie with a 2.5 hour run time.