

Slope Interpretations in Multiple Regression Models

Our fitted multiple regression equation relating the average bill depth of Antarctic penguins (y_i , measured in mm) to their bill length (x_{i1} , measured in mm) and flipper length (x_{i2} , measured in mm) is

$$\hat{y}_i = 34.31 + 0.09(x_{i1}) - 0.11(x_{i2}).$$

1. Use this fitted regression line to estimate the average bill depth in four different subpopulations of penguins:
 1. **Population A:** penguins with a bill length of 35 mm and a flipper length of 212 mm
 2. **Population B:** penguins with a bill length of 45 mm and a flipper length of 220 mm
 3. **Population C:** penguins with a bill length of 45 mm and a flipper length of 212 mm
 4. **Population D:** penguins with a bill length of 55 mm and a flipper length of 220 mm

2. The penguins in Populations **B** and **C** have a bill length that is 10 mm longer than that of the penguins in Population **A**; similarly the penguins in Population **D** have bill lengths that are 10 mm longer than that of the penguins in Populations **B** and **C**. Use your answers from question 1 to estimate:
 - The difference in average bill depth between penguins in population **B** and population **A**
 - The difference in average bill depth between penguins in population **C** and population **A**
 - The difference in average bill depth between penguins in population **D** and population **B**

3. Based on your answers to question 2 above, critique the following possible interpretation of the slope coefficient for bill length in our multiple regression model: “A 10 mm increase in bill length is associated with a 0.9 mm increase in the average bill depth.”

4. Based on your answers to question 2 on the previous page, critique the following possible interpretation of the slope coefficient for bill length in our multiple regression model: “A 10 mm increase in bill length from 35 mm to 45 mm is associated with a 0.9 mm increase in the average bill depth.”

5. Based on your answers to question 2, critique the following possible interpretation of the slope coefficient for bill length in our multiple regression model: “For penguins with a flipper length of 220 mm, a 10 mm increase in bill length is associated with a 0.9 mm increase in the average bill depth.”

Bonus questions

1. What is the minimum amount of information we need to know to calculate the estimated mean bill depth (or, relatedly, make an individual prediction) using our fitted multiple regression model? *Circle all that apply.*
 - a. The exact value of the penguin bill length (x_1)
 - b. The exact value of the penguin flipper length (x_2)
 - c. That penguin bill length is held constant
 - d. That penguin flipper length is held constant
 - e. That penguin bill length increases by one
 - f. That penguin flipper length increases by one
2. What is the minimum amount of information we need to know to calculate the estimated change in mean bill depth associated with a particular increase in penguin bill length? *Circle all that apply.*
 - a. The exact starting value of penguin bill length (x_1)
 - b. The exact value of the penguin flipper length (x_2)
 - c. That penguin bill length is held constant
 - d. That penguin flipper length is held constant
 - e. The amount by which penguin bill length increases
 - f. The amount by which penguin flipper length increases