

Visualizing the Additive Model

The fitted additive model for the probability of surviving the Donner Party expedition, modeled as a function of both age and sex, is

$$\text{logit} \left\{ \hat{\pi}(\text{age}_i, \text{male}_i) \right\} = 1.622 - 0.036 (\text{age}_i) - 1.068 (\text{male}_i).$$

1. Sketch a graph of the fitted regression equation on the log odds scale. Clearly label your x and y axes, provide specific numbers for salient features of the graph (such as the y -axis intercept and slope), and indicate which parts of the graph correspond to male members of the expedition and which parts correspond to female members of the expedition.
2. What do you think this same fitted regression equation looks like on the probability scale? Create a rough sketch representing your best guess!

Interaction Models

The population form of our interaction model of the Donner Party survival data is

$$\text{logit} \left\{ \pi(\mathbf{age}_i, \mathbf{male}_i) \right\} = \beta_0 + \beta_1 (\mathbf{age}_i) + \beta_2 (\mathbf{male}_i) + \beta_3 (\mathbf{age}_i \cdot \mathbf{male}_i)$$

1. What is the corresponding population model for the log odds of survival among men on the expedition? What about among women?
2. Use the following code to read in the Donner Party data. Then, fit the interaction model above. Write down the fitted logistic regression equation.

```
donner <- alr4::Donner
```

3. What is the estimated odds ratio for the association between age and survival among men? What about the estimated odds ratio for the association among women?
4. Suppose we want to use the interaction model to test the hypothesis that sex is associated with survival, even after accounting for age. Write down the null and alternative hypotheses for this test in two ways: in terms of the restrictions being placed on the model coefficients (i.e., which β s are being set equal to zero?) and in terms of the full and reduced population models being compared.
5. Fit the full and reduced models from part 5 and report the deviance for each of these models.
6. Conduct the corresponding likelihood ratio test at the $\alpha = 0.05$ level and report the your results of your test in context.